Test on probability, and graphs on Monday, November 30, 2015.

**Graphs**

**Terminology:**

1. Degree –
   * In-degree –
   * Out-degree –
2. Path –
   * Length of path –
3. Cycle –
4. Acyclic –
5. Adjacent vertices –
   * Neighbors --
6. Incident vertex – a Node’s out node(s)
7. Bipartite graphs –
8. Complete [bipartite] graphs –
9. Simple graph – no self loop

**Modeling an airline’s routes**

* Vertices are airports
* Edges are non-stop flights
* Edge cost: cost of a ticket or distance

*What is the shortest path from one node to another*

*Strong connectivity is desired*

**Modeling traffic flow**

* Vertices are street intersections
* Edges are streets
* Edge cost: distance/speed limit/congestion/etc.

**Modeling a social network**

* Vertices are people
* Edges are “friend”/”follow” links
* Edge cost: number of messages exchanged/similar likes/etc.
* “All vertices reachable by paths of length ≤ k

**Theorem**: The sum of the degrees of all the verities in *G = (V,E)* is *2* x |*E*|

Prove the statement, “*There is a group of nine people in which each is friends with exactly five others*” is false

**Corollary:** The sum of the degrees of all the vertices in a graph is even

**Corollary:** In any graph, there are an even number of

*In a group of two or more people, must there always be at least two people who are acquainted with the same number of people within the group? Why?*

1. In a simple graph, must every vertex have a degree that is less than the number of vertices in the graph? Why?
2. Can there be a simple graph that has four vertices each of different degrees?

Suppose *r* and *s* are any positive integers. Does there exist a graph, *G*, with the property that *G* has vertices of degrees *r* and *s* and no other degrees? Explain.

Representation of graphs

1. Adjacency Matrix – A |v| x |v| matrix, with the [i,j]’th entry representing the edge from the i’th to the j’th vertex

* Weighted graph: the matrix entries denote the edge-weights
* Some sentinel value (depends on application) for non-existent edges

1. Adjacency List – an array of linked lists of length |v|, with the i’th entry denoting the edges from the i’th vertex

**Topological Sort**

Given directed acyclic graph (DAG), *G*=*(V,E).*